Statistical Fatigue and Residual Strength Analysis of New/Aging Aircraft Structure

Scott Fawaz
US Air Force Academy (USAFA), Colorado Springs, CO
scott.fawaz@usafa.edu

Börje Andersson
The Swedish Defence Research Agency FOI
ba@foi.se

1. Introduction

State-of-the-art fatigue life prediction algorithms can only consider rather simple structural cracking problems because numerical data for complex multiple crack configurations are simply not available. Unfortunately modern aircraft are complex assemblies with diverse materials and joining methods. As a result, mission planners and combat leaders are forced to use much simplified approaches which results in over- or un-conservative predictions which might lead to costly decisions and/or jeopardize aircraft safety. By use of novel mathematical-numerical methods and the HPCMP computational resources, the situation can be drastically improved resulting in better fleet management in peace- and war-time. In the present work the world largest stress intensity factor database is being populated with high-accuracy solutions. Old low accuracy solutions are also identified and replaced. Today, the database size is about two orders larger than that available in 2002 for fatigue crack growth predictions. This is a continuing work. The data base created applies to both civil and military aircraft. Methods and data developed will allow the mission planner to assess the level of risk associated with future missions with respect to how damage accumulates per flight. The present work also covers residual strength analysis of the center wing box (CWB) of the C-130 aircraft for in-service observed crack configurations. This very challenging task is briefly summarized below.

The present paper gives details with regard to creation of the stress intensity factor data base. In addition we describe the enabling numerical-mathematical technologies for residual strength analysis of a full C-130 CWB where all geometrical details are modeled as three-dimensional (3D) objects down to rivet/bolt size and nonlinear behavior captured with control of error in the numerical solution.

2. Worlds Largest *K* Database

The work consists of deriving 92.4 million stress intensity factor solutions for two cracks located at two holes in a plate (Figure 1) for various *R/t, a/t, a/c, D1/D2, L*, and *B*, for through thickness cracks and not-through thickness cracks at different crack locations.

The splitting scheme described in the UGC2006 lecture[1] was adjusted to the case of two cracks with crack fronts that might almost be touching. Figure 1 shows schematically the three different solutions to add in order to derive the solution of interest. Each single crack problem (totally about 68,000) is analyzed separately. From the single crack solutions the, 92.4 million solutions can be obtained with control of the error as described in Reference 1 and references therein. The global (crack free) problems are analyzed for specially designed load cases where traction loading and displacement jumps are described on arbitrary surfaces which must not intersect the crack surfaces. Figure 2 shows one such surface where tractions are applied. Depending on the crack sizes, different loading surfaces have to be used since the loading surface must not intersect a crack surface. Hence each one of the 68,000 single crack problems must take into account a hierarchy of load surfaces which depend on the multiple crack cases of interest.

2.1. Effectiveness and Robustness

The six-level computational scheme[1] which is used to derive the 92.4M solutions has to be equipped with automatic functions at most levels if manual work should not be prohibitively large. Examples of automatic functions that have been developed are mesh generation (68k meshes for the *hp*-version of FEM), job submission(>68k jobs), job monitoring functions, scrubbing/resubmitting, data storage, data transfer etc.

Submission of thousands of small 4-8 processor jobs prevents efficient optimization of the job environment to be performed by the system load scheduler. Various job clustering techniques have therefore been developed in order to support system optimization, and throughput of the 68,000 jobs. Significant efforts (i.e. direct simulation)

are also being made to check that the 92.4M solutions will be derived with a point wise error less than 1%.

3. Residual Strength of the C-130 Hercules Aircraft Center Wing Box

Residual strength analysis of the C-130 CWB requires state-of-the-art damage and fracture analysis tools combined with software and hardware for true large-scale analysis. The *hp*-adaptive code STRIPE which has unique capabilities for multi-scale damage and fracture mechanics analysis is used for this purpose. STRIPE uses mixed OpenMP and MPI and executes efficiently on the IBM P4 system KRAKEN/BABBAGE at NAVO and at the SGI Altix system EAGLE at ASC.

The main questions to be addressed are if the C-130E/H aircraft now being retired have sufficient residual strength at retirement with respect to fatigue cracking in the CWB. In other words, was the aircraft in service too long? Conversely, does the aircraft have adequate residual strength at retirement which would allow safe usage for a significantly longer time; i.e., aircraft might have been prematurely retired.

The development part is related to solution of very large nonlinear problems of the size of billions of degrees of freedom (GDOFs) with control of the numerical error in the solution. Aircraft structural mechanics problems of this complexity have hitherto never been solved. Problems of such complexity remain a target in industry for the next decade in order to do more virtual and less physical testing.

An important aspect of the multi-scale scheme used is efficient handling of problems with several 100,000 right hand sides[1] for problems of GDOF size. Significant code rewriting, load balancing algorithms and code optimization have so far resulted in an efficiency which for problems of size 0.2 GDOFs can be expressed as:

$$T_{LC} = T_1 *(1+LC/f), \qquad (1)$$

In Eq. (1) we have today $f=4,000$. $T_{LC}(p)$ is the wall time needed to solve the problem with LC right hand sides and T_1 the wall time needed to solve the problem with one right hand side. By using the MPI-IO extensions for parallel I/O introduced in the MPI-2 standard the factor f can most likely be significantly increased.

Figure 3 summarizes corrosion damage and fatigue cracking found in a retired C-130 aircraft. Cracks were found after a complete tear down process where all surfaces were inspected for cracks and corrosion damage. Figure 3 shows six of the over 40 fatigue critical locations in the wing box and also regions with extensive corrosion.[2]

Figure 4 shows the structural domain currently being meshed at the US Air Force Academy. A stringer detail is shown in order to exemplify the mesh density used. The mesh generator TrueGrid™, developed by XYZ Scientific is used to create the large finite element mesh which is estimated to consist of about 25 million finite elements. Derivation of accurate solutions for local stresses, including control of the error in the solution will require solution of problems of a size 1–2 GDOFs. Structural problems of this size are about two orders larger than the largest structural mechanics problems being solved in aircraft industry today.

Efficient solution of 1–2 GDOFS problems with several 100,000 right hand sides constitutes a true challenge. Improved domain decomposition division schemes, load balancing algorithms, memory/cache utilization and general software improvements in order to handle FE-meshes consisting of about 25 million elements has been developed.

Several generic C-130 models of increasing sizes were created and successfully analyzed on the ASC/eagle system. Figure 5 shows a mesh of a generic C-130-wing consisting of 7 million hexahedral finite elements. A mesh twice that large was solved for polynomial orders *p*=3 having 418 MDOFs. This problem might be the largest aircraft structural mechanics problems ever solved. The work continues to reach objective of efficient solution of problems of GDOF-size. Difficulties encountered are in many cases believed to be related to compiler errors and use of proper environment parameters like stack size limit etc.

Table 1 summarizes results from some test computations with increasingly larger meshes (only 64 processors were used in the tests). Good scaling up to 512 processors has been demonstrated.

Table 1. Benchmark test examples executed at ASC/EAGLE (SGI Altix) using 64 processors and 256 GBYTE of memory

Model	Elements	P	MDOFs	Wall time (h)
Part of Fuselage, Fig. 6	1M	2	17	1.4
-"-	-"-	4	53	5.1
-"-	-"-	6	102	27
½ Generic C-130, Fig. 5	7M	2	122	8
-"-	- "-	3	209	15
-"-	-"-	4	371	44
1/1 Generic C-130	13M	2	242	30
-"-	-"-	3	406	55

3.1. Error Estimation

Despite the high mesh density used, the error in local stress has somewhat surprisingly been found to be as large as 15% in local areas, if 20 nodes hexahedral finite elements are used. Figure 6 shows a converged nonlinear

solution for p=6 (105-noded hexahedral elements) having a total of 102 MDOfs. The figure shows large global buckles interacting with local buckles. Near local buckles, red areas in the plot in Figure 6, buckles that might contribute to de-bonding between stringer and skin and/or rivet failure can be caused by the 15% error in the p=2 versus p=6 solutions.. The hp-version of FEM used in the present project makes the necessary control of the error in the solution for local quantities of interest possible.

3.2. Accurate and Fast Solution of 3D Contact Problems

The residual strength analysis of the C-130 CWB requires reliable calculation (i.e. error control) of the maximum load carrying capacity of the wing box for a given multi-crack scenario with interacting cracks, buckling, failing rivets. We here briefly indicate the strategy used to solve the contact problem between rivets/bolts and the aircraft skin/stiffeners. Due to the nonlinear character of the contact problem an iterative scheme has been designed which is both robust, virtually exact, converges fast and simultaneously takes the mathematical character of the problems appearing during the iteration steps into account. The contact problem (see Figure 7 for an example in a two-dimensional (2D setting) of interest is split into three problems, similarly as shown in Figure 1. The global problems a) and c) in Figure 7 are analyzed without considering either the contact surfaces or the crack surfaces, i.e., all surfaces are in perfect sliding-free contact. The splitting scheme used then has the advantage that the global analysis a) and c) which is very costly for problems of GDOF-size do not have to be repeated for varying crack sizes and contact surfaces. The local problems b), see also Figure 8 are analyzed for fixed (but *a priori* unknown) locations of the contact surfaces for a set of loading functions defined by carefully selected traction basis functions $Q_1 - Q_4$ and scaling factors $\beta_{(m,I)}$, the primary unknowns.

A necessary condition to be satisfied at the two points separating contact and no-contact is:
$$K_I = 0 \tag{2}$$
and that normal stresses are compressive at the part of the circular boundary being in contact. Equation (2) can be used as a very precise criterion to find the two points separating contact and no-contact. A simple Newton-type algorithm can then be used to find the contact points satisfying Eq. (2) for each bolt/rivet. The splitting scheme is simply used as a very fast solver for determination of stress intensity factors at the crack-tips and at potentially correct contact positions during iterations. Note that during iterations and before convergence is achieved $K_I \neq 0$ implying that there is a strong stress singularity at the assumed contact positions.

Robustness of the approach is obtained by using a hp-type of mesh moving with the location of the assumed contact points. Experience shows that the algorithm converges to about 3 digits accuracy in less than 5 iterations. Figure 9 gives an overview of the contact solution scheme in a 3D setting.

3.3. Rivet Failure

Rivets are normally not loaded in tension but might see tensile loads in post-buckling regime, especially in case of shear buckling but also in presence of (larger) cracks. The model used to simulate bolt/rivet failure is based on a cohesive zone type of model resting on an assumption that all deformation takes place in a plane in the bolt. Figure 10 exemplify the location of the cohesive zone in case of a bolt. The adaptive mesh design based on the hp-version of FEM which follows the contact lines are also indicated in Figure 10. Normal and shear tractions (σ, τ) are assumed to depend on the displacement jumps over the cohesive zone. In the spirit of the above mentioned schemes for solution of crack problems and contact problems a modified splitting scheme is used to simulate rivet/bolt failure due to shear and tensile loading. This approach maintains the robustness, accuracy and efficiency of the overall approach and makes statistical analysis economically feasible.

4. Summary

The focus of the present paper is the development of advanced computational methods for reliable fatigue and residual strength analysis of bolted-riveted-bonded metallic structures. The main steps discussed are:

- Creation of the extended data base where this part will consist of 92.4M solutions
- Fast solver for solution of GDOF-problems with several 100,000 right hand sides
- Creation of FE-meshes of the entire C-130 main wing box
- Benchmarking on generic C-130 models with up to 0.4 GDOFs
- Development of a splitting scheme for simulation of rivet/bolt failure has been initiated

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References

1. Andersson, B. and S. Fawaz, "Statistical Fatigue and Residual Strength Analysis of New/Aging Aircraft Structure." UGC 2006.
2. Shoales, G. and P. Christiansen, "C-130 Center Wing Box Structural Teardown Analysis Program," Proc. at the 24th International Committee on Aeronautical Fatigue Symposium, Naples, Italy, 16 18 May 2007.

$$\overline{U} = U_G^{(0)} + \sum_{m=1}^{M}\sum_{l=1}^{L}\alpha_{(m,l)}U_G^{(m,l)} - \sum_{m=1}^{M}\sum_{l=1}^{L}\alpha_{(m,l)}U_L^{(m,l)}$$
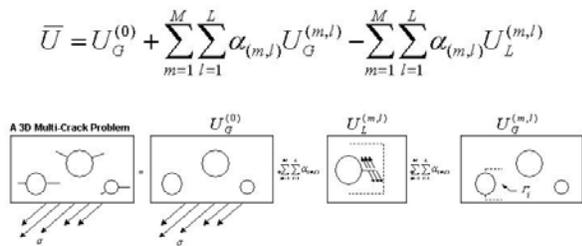
Figure 1. Schematic picture of splitting scheme in a 2D setting and in case stress intensity factor calculation for the case with known contact surfaces (see Reference 1)
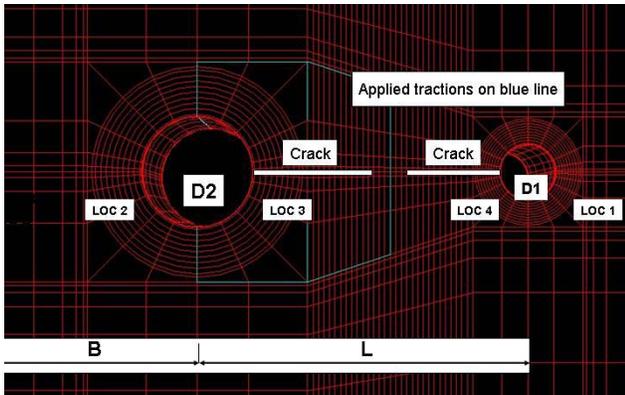
Figure 2. Definition of parameters *L,B,D1,D2* and crack locations 1–4. Two cracks at locations 3 and 4 respectively are shown together with a proper path (blue color) for load/displacement jump application for this crack combination.

Center Wing Box FCL focus from CWB Tear Down

Areas of Interest:
• Fatigue Critical Locations: CW 1, CW 5C, CW 7B, CW 9, CW 14, CW17
• Corrosion Indications: SAIC beam cap corrosion inspections
• Crack Indications: SAIC FWD lower beam cap and all rainbow fitting inspections for cracks
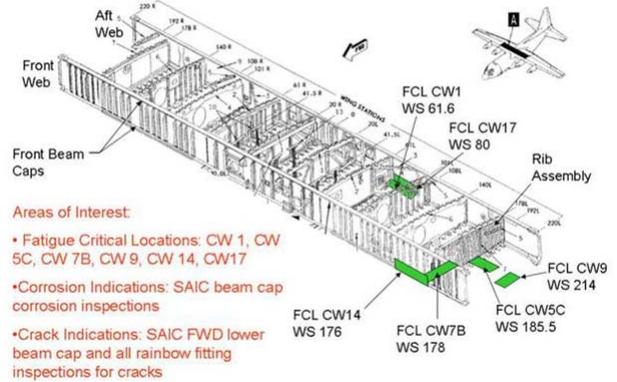
Figure 3. Fatigue and corrosion sensitive regions found after tear down and inspection of a C-130 wing box

Figure 4. Center C-130 wing box which currently is being meshed in large detail at USAFA. Stringer shown in the figure exemplifies a characteristic mesh density.
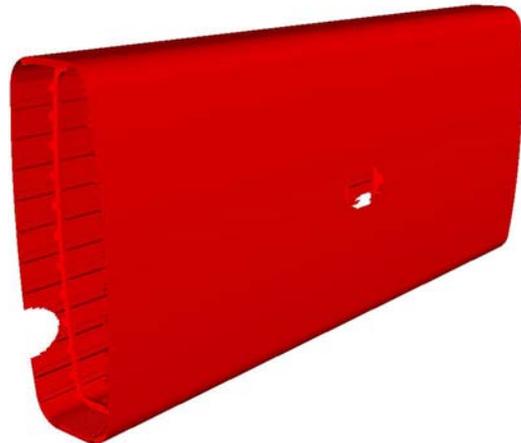
Figure 5. Generic ½ mesh of C-130 wing box consisting of 7 million hexahedral elements
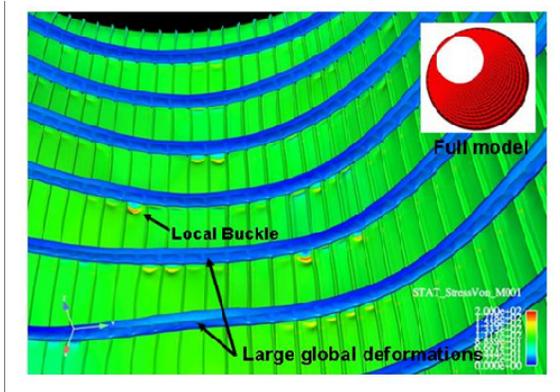
Figure 6. Interacting local and global buckles in generic fuselage. The converged solution shown is the p=6 solution having 102 MDOFs.
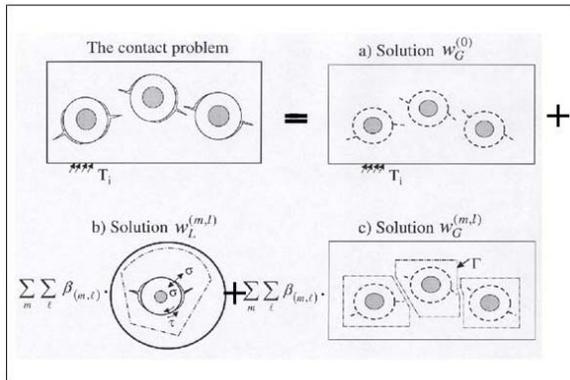

Figure 7. Schematic picture of splitting scheme in a 2D setting and in case stress intensity factor calculation for the case with *unknown* contact surfaces
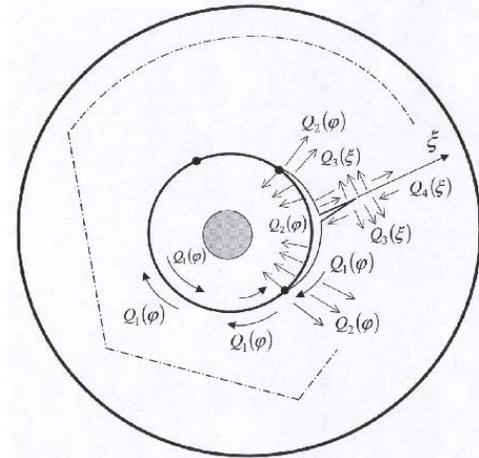

Figure 8. Loads on local problems are shear tractions $Q_1(\varphi)$ on the entire circular boundary, normal tractions $Q_2(\varphi)$ on the part of the circular boundary not in contact, shear tractions $Q_3(\varphi)$ and normal tractions $Q_4(\varphi)$ on the crack face, respectively. All loading cases are treated separately.
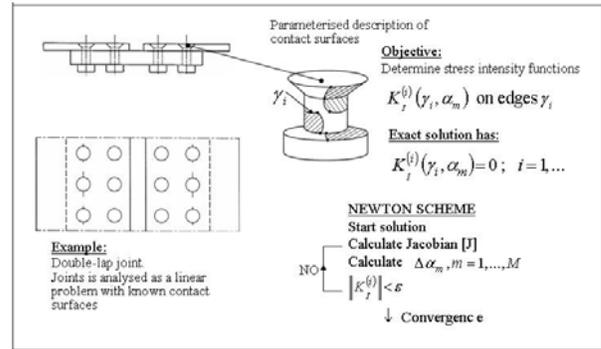

Figure 9. Steps in solution of 3D contact problems using a fracture mechanics approach to find contact areas and crack data with high accuracy in a few iterations
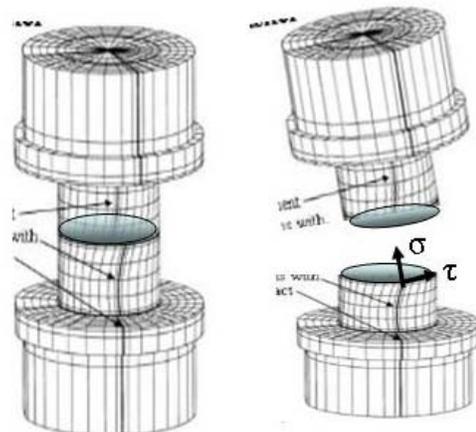

Figure 10. Cohesive zone model used for robust, accurate and efficient simulation of bolt/rivet failure