

Stress Intensity Factors and Crack Interaction of Part-Elliptical Through Cracks in Adjacent Holes

J.J.M. de Rijck^{*}

S.A. Fawaz[†]

J. Schijve[‡]

A. Vlot[‡]

Cracks nucleating and growing from adjacent fastener holes are common in transport aircraft fuselage structure. To study this phenomenon, both analytical and experimental investigations were completed. In the experimental part, a better understanding of the crack shape development and crack growth behavior of cracks emanating from adjacent holes is achieved by using a combined tension and bending specimen. From the experimental results, the crack growth rate, crack shape and crack interaction effect for different initial crack shapes are determined. In the analytical part, stress intensity factor solutions for equal-sized, part-elliptical through cracks growing from an array of holes subject to remote tension, remote bending, and pin loading were calculated. The three-dimensional virtual crack closure technique is used to calculate the new K solutions. It is shown that oblique part-elliptical through cracks do not interact until very late in their fatigue life. This effect was more pronounced for large shallow cracks. In other words, straight through cracks interact earlier than oblique through cracks.

1 INTRODUCTION

Metal fatigue was the unseen danger that brought down the first commercial jet airliner. On 10 January 1954, the Comet G-ALYP crashed into the Mediterranean Sea. It took an extensive test program on the Comet to reveal the cause of the accident. It was shown that initiation of small cracks at a highly stressed area around a window panel resulted in small cracks growing into a row of rivets causing the fuselage to fail when pressurized in flight. This shows that as early as 1954 aircraft designers have had to cope with metal fatigue.

Cabin pressurization in a transport aircraft fuselage creates the primary loading condition, namely a tensile membrane stress (hoop stress). This hoop stress, see [Figure 1](#), creates a bending stress due to the joint eccentricity and a bearing stress due to the load transfer through the fasteners. [Figure 1](#) shows a simple longitudinal lap joint of the fuselage skin. The dominant load cycle of the fuselage is the ground-air-ground (GAG) pressurization cycle, which can cause fatigue problems for fuselages.

Past and present investigations of the behavior in lap-splice joints have provided information on the characteristic fatigue crack shape found in aircraft riveted lap-splice joints. In [ref \[1\]](#), tests were performed using thin sheet 2024-T3 clad aluminum alloy with a centrally located hole subjected to remote tension and bending. These investigations clearly showed the influence of bending on the overall fatigue life and crack-front shape development. The purpose of the

^{*} Netherlands Institute for Metals Research, Faculty of Aerospace Engineering, Delft University of Technology, Delft, The Netherlands

[†] Air Force Research Laboratory, Wright-Patterson AFB, Ohio, USA

[‡] Faculty of Aerospace Engineering, Delft University of Technology, Delft, The Netherlands

experimental part of the current effort [2] is to show the degree of crack interaction between two oblique through cracks growing toward one another from adjacent fastener holes. To investigate the degree of crack interaction, new tests are performed on thin sheet 2024-T3 clad aluminum alloy using similar test specimens as used in [1]. The centrally located hole is replaced by two adjacent holes. From these tests, an estimate of the fatigue life is obtained. In addition, since multiple tests are done, an indication of scatter in the fatigue life is also obtained. Observations of the crack interaction are obtained when measuring the crack growth visually during a fatigue test which shows the crack growth during its life. When using a fatigue spectrum that leaves marks, groups of distinct fatigue striations on the fracture surface, the crack history can be reconstructed using a scanning electron microscope (SEM). The SEM investigation can show the entire fracture surface and usually the location where the crack initiated. Most importantly, a more accurate crack growth curve can be obtained.

Earlier research [1] showed that the three-dimensional virtual crack closure technique, 3D VCCT, is a useful tool for calculating stress intensity factors for specific crack front shapes and load conditions commonly found, not only in laboratory test specimens, but also from in-service experience. The 3D VCCT procedure as described in ref [1] is also used in the analytical part of the research for calculating new K solutions. Such solutions are not yet available for cracks growing from adjacent holes toward one another through the ligament length between the holes. Before generating the part-elliptical through crack solutions, a verification study has been performed to check that the finite element model is of sufficient fidelity, ref [2]. Finally, K calculations are made for complex loading conditions representing axial tension, secondary bending, and pin loading. To investigate the influence of pin loading, three different pin load distributions were used in ref [1], namely a concentrated load, a cosine and cosine² pressure distribution in the bore of the hole. From the results in ref [1], the cosine² pressure distribution appears to be the most appropriate and is thus used in this investigation.

2 EXPERIMENTAL INVESTIGATIONS

Riveted fuselage skin panels of transport aircraft experience a complicated stress system due to the inherent structural complexity of the joint. Decomposing the total stress into simpler, easier to understand load conditions makes the problem tractable. The remote tension load condition has been investigated thoroughly and is well understood. Secondary bending is a more complex load condition, introduced by eccentricities of the joint and curvature of the fuselage skin. Therefore, secondary bending is investigated in this part of the study. Specifically, an investigation on the influence of secondary bending on open (rivet) holes for longitudinal joints is completed. However, influences of the rivets are neglected, such as residual stresses due to the rivet installation, fretting corrosion between the faying surfaces, but also between the rivet and rivet hole, and load transfer due to friction between the faying surfaces. The influence of secondary bending is investigated by using a specially developed combined tension and bending fatigue specimen [1,3,4].

2.1 FATIGUE SPECIMENS

All specimens are manufactured from one sheet of 2024-T3 clad aluminum alloy with thickness $t = 2.0$ mm. Using only one sheet should help minimize scatter in the results [5]. Figure 2 shows the combined tension and bending fatigue specimen. The specimens have a TL orientation just as

in fuselage skins of transport aircraft. It means that the crack grows in the same direction as the rolling direction and is perpendicular to the applied remote stress. In [5], it is shown that a reduction in fatigue life of 30% for 2024-T3 Alclad could be observed between LT and TL directions.

The location of the holes is shown in Figure 2, the rivet hole diameter $d = 5.10$ mm and the hole pitch $s = 25.4$ mm. The position of the holes occurs at the location of the maximum bending stresses in the specimen. It is of major interest to know the magnitude of the tension and bending stress at the position of the two holes. To verify the calculated stresses a static test has been conducted with ten strain gages. Figure 3 shows the position of the strain gages at the center of the specimen. Five strain gages are bonded on each side of the specimen, two strain gages perpendicular to the direction of the remote loading and three strain gages in longitudinal direction. The two strain gages perpendicular to the loading direction are used to measure the influence of the Poisson contraction. The results are shown in Figure 4 [3]. Note the good correlation between the measured strains and the calculated results of a one-dimensional analytical line model [1,4].

A number of specimens were provided with artificial cracks at both sides of each hole. Four crack shapes were adopted for through cracks as shown in Fig. 5. These shapes correspond to those seen in service and thus are also used in the FEA discussed in the subsequent section. The initial cracks are manufactured by electric discharge machining. An EDM electrode with a thickness of 0.15 mm was used, which leads to an EDM notch thickness of 0.2 mm. The shapes used for the electrodes are based on the elliptical shapes shown in Figure 5, with c_1 as the crack length along the surface, which would be the faying surface in a riveted joint, and c_2 as the crack length at the opposing free surface. One small part through crack shape is used, with a circular shape with a radius of 0.1 mm, see Fig 5.

2.2 CRACK MEASUREMENTS

During the fatigue tests, an estimate of the fatigue life for that specimen can be obtained, and since multiple tests are done, an indication of the scatter in fatigue life can also be found. Examination of *in situ* crack growth measurements can provide insight into the crack growth behavior. At the very least, the fatigue life, crack growth history, and crack growth rate can be quantitatively determined. When using a fatigue spectrum that leaves marks on the fracture surface, the crack history can be reconstructed using a scanning electron microscope. The use of marker loads, typically a instantaneous or short duration variation in the CA maximum stress or stress ratio, can perturb the striation spacing created by the CA loading. If the perturbations can be reliably detected in the electron microscope, the crack growth history can then be determined if the number of cycles to failure, also known as the fatigue life, is known. The SEM investigation can show the entire fracture surface and many times the exact location where the crack initiated. Most importantly, a more accurate crack growth curve can be obtained.

Cracks in lap-splice joints nucleate as corner or surface cracks near the rivet hole at the faying surface of the joint. After nucleation and due to cyclic loading, cracks continue to grow through the thickness towards the free surface of the joint, opposite of the initiation site. As a result of the loading condition, the crack continues to grow as a part-elliptical through crack once it penetrates through the thickness. As explained before, the primary loading condition is the hoop stress. The

hoop stress will create a bending stress due to the joint eccentricity and bearing stress due to the load transfer through the rivets. This part-elliptical oblique crack shape is maintained throughout the life of the joint even when adjacent cracks start to interact with one another. This interaction causes an increase in the crack growth rate ultimately resulting in crack link-up at the faying surface.

In situ visual crack detection was done with an optical microscope and a ruler with an accuracy of 0.25 mm. The crack length was measured at cycle intervals dependent on the crack growth, i.e. larger intervals when the crack size was small, and smaller intervals later on when the crack growth rate was high. These measurements are commonly shown as crack length versus fatigue life curves (a vs. N curves, [Figures 11 and 12](#)).

Some specimens were tested under constant amplitude (CA) loading until a certain crack length was reached, and then pulled statically until failure. A clear crack front could then be observed on the fracture surface. The coordinates of the crack front were measured with an optical microscope equipped with a linear voltage displacement transducer (LVDT) attached to the stage. [Figure 6](#) shows crack fronts from five separate specimens.

Other specimens were fatigue tested until failure. In these tests, a marker spectrum was used to create marker bands, groups of fatigue striations in a particular sequence, detectable in the SEM to determine the crack size and shape during the fatigue test. Two spectra were used, a marker load (ML) spectrum and an overload (OL) spectrum, [Figures 7 and 8 respectively](#). The ML spectrum does not cause crack growth retardation, which may complicate fatigue life predictions; conversely, the OL spectrum does cause retardation [\[1,2,6\]](#). However, with the OL spectrum, it is easy to reveal the crack shape development during the fatigue life. The OL spectrum creates visible bands with larger striation spacings, which are easily detectable on the fracture surface. [Figure 9](#) clearly shows that the ML spectrum results in similar crack growth rates as a constant amplitude loading. For a discussion on how the crack growth rates are affected by an OL spectrum, see ref [\[1,2,6\]](#).

Marker bands are difficult to detect when the applied load is high because marker band formation is disrupted by micro-voids leading to a rather tortuous fracture surface. Marker bands were still found on the fracture surface of specimens tested with an applied remote stress of 100 MPa (applied load of 20 kN), see [Figure 10](#). However, a complete crack growth history could not be obtained since marker bands were only found for small crack sizes, [Figure 11 and 12](#). Using an applied remote stress of 75 MPa (applied load of 15 kN) results in a lower stress at the area of interest (minimum net section between the two holes). [Figure 4](#) shows that a remote stress of 100 MPa leads to a bending factor $k = 1$ at the area of interest, where k is defined as:

$$k = \frac{\sigma_{\text{Bending}}}{\sigma_{\text{Tension}}} \quad (1)$$

In this case the bending and tensile stress are both approximately 100 MPa, which implies a stress of 200 MPa at the faying surface and 0 MPa at the free surface. As mentioned before, a high stress prohibits marker band formation and detection, thereby limiting a reconstruction of the complete crack growth history. To facilitate marker band creation, a somewhat lower maximum load of 15 kN was applied. According to [Figure 4](#), $k = 1.66$ with a tensile stress at the faying surface of 133 MPa and a compressive stress of 33 MPa at the free surface. Even with this

lower applied load, the fracture surface remained quite tortuous. A further complication appeared on the free surface due to the compressive stress, the fracture surfaces then come into contact which destroys the marker bands.

Because the OL spectrum influences the fatigue life of the test specimens, the results could only be used for crack shape determination. Similar crack front shapes were observed as found in the other tests with the ML spectrum.

3 FINITE ELEMENT ANALYSIS

A number of different methods are available to obtain stress intensity factors for cracked three-dimensional bodies. Most of these methods are difficult and therefore require an experienced analyst. Stress intensity factors can be derived from finite element analysis results by either direct or indirect methods [1]. Direct methods use the FEA results to calculate K explicitly. Indirect methods use the FEA results to calculate an intermediate parameter which in turn can be used to calculate K . In the 3D VCCT, the FEA results are used to calculate the strain energy release rate which is then used to calculate K . Using the 3D VCCT avoids most of the difficulties encountered by other methods, e.g. neither singular elements at the crack front nor creating elements normal to the curved crack front are required. In order to investigate the interaction of cracks from adjacent holes, a plate with an infinite row of holes is considered. The same oblique cracks occur at both opposite edges of the holes. In view of symmetry, the model to be analyzed has a geometry as shown in Figure 13. An additional symmetry plane is used at $x = b$. The model consists of 7,712 eight noded solid isoparametric elements with 9738 nodes resulting in 29,214 unconstrained degrees of freedom. Since the mesh around the crack does not require special elements, only one finite element mesh is used for all K calculations. Further information on the procedure used can be found in ref [1]. The boundary conditions are: At $x = 0$, the u displacements are constrained, and at $y = 0$, the v displacements are constrained, where u , v , and w are the displacements in the global (x , y , z) directions. At $x = b$, all u displacements are constrained, i.e. Poisson contractions are constrained. When applying a remote unit stress ($\sigma = 1$) at $y = h$, the $u = 0$ constraint at $x = b$ introduces a non-uniform stress σ_{xx} since the fixed nodes at $x = b$ prevent the Poisson contraction in the xy -plane.

The σ_{xx} gives a similar stress state at $x = b$, as presented by Schulz for a 2-dimensional infinite width plate with array of holes [7]. Furthermore, Broek showed that for an infinite row of collinear cracks, the same stress distribution at the edges ($x = b$) could be found [8]. A finite element analysis showed a difference (8%) between the 2D analytical stress concentration factor solution by Schulz and the present solution. A difference is to be expected when comparing solutions for small through cracks at the bore of the hole for a single central hole and an infinite row of holes. Since the crack interaction is of major interest and the small differences introduced by σ_{xx} are negligible for larger cracks, the crack interaction solutions are therefore not affected by the boundary conditions.

Three different loading conditions are applied, an axial unit tensile stress (at $y = h$), a remote unit bending stress (at $y = h$ which varies linearly through the thickness) and a pin load (cosine squared pressure applied to the bore of the hole). The dimensions shown in Figure 13 are $h = 76.2$ mm, $b = 12.7$ mm and $r = 2.54$ mm. The height $h = 76.2$ mm is chosen to eliminate any finite height effects [9].

4 FINITE ELEMENT ANALYSIS OF INTERACTION BETWEEN CRACKS EMANATING FROM ADJACENT HOLES

The stress intensity factor solutions are calculated for frequently observed crack shapes, see [Table 1](#). [Figure 5](#) and [Figure 14](#) show these crack shapes. The shapes and also the loading conditions, see [Figure 13](#), are similar to those used in [ref \[1\]](#). In the present analysis a comparison will be made between the stress intensity factor solutions for cracks emanating from a single hole and from an infinite row of holes.

The calculated K's are normalized as shown in Eqn. (2) to yield the geometric correction factor, β .

$$\beta = \frac{K_I}{\sigma\sqrt{\pi c_1}} \quad (2)$$

For all load cases, K is normalized in this manner where σ is the applied remote stress and c_1 is the crack length at a given z/t . Note, since the model represents an infinite sheet, no finite width correction is included in the normalization of K.

Trends and calculated β 's for oblique part-elliptical through cracks emanating from a single hole can be found in [ref \[1\]](#). Trends and the newly calculated β 's for oblique part-elliptical through cracks emanating from adjacent holes have been presented in [ref \[2,10\]](#). These new K values show in general the same trends as observed in [ref \[1\]](#) for a single hole. However, some obvious differences are found due to the interaction of cracks growing towards one another.

Figure 14 shows a comparison between the β -values for cracks at an infinite row of holes and cracks at a single hole for $a/t = 1.05$ and $r/t = 1.0$. For the smallest crack (case 1, $a/c_1 = 10$) the β -values for a single hole are slightly larger than for a row of holes. This is a result of a different boundary condition adopted in [1] for a single hole; instead of a restrained Poisson contraction, a plate with a non-uniform normal stress σ_{xx} at $x = b$ was analyzed. This normal stress lowers the β values for cracks with large values of a/c_1 (small cracks with a more straight crack front), and generally results in a slightly lower β along the entire crack front. An indication of crack interaction for the present study can be found for larger shallow cracks (smaller a/c_1 values and $c_2 \ll c_1$). For these cracks, higher β solutions are calculated for the present investigation, which indicates an interaction between the two cracks from adjacent holes. From [Figure 14](#), it can be seen that for $a/c_1 = 0.6$ the first indications of crack interaction can be seen through slightly higher β solutions. With $a/t = 1.05$, $r/t = 1.0$ and $a/c_1 = 0.6$, the crack length is $c_1 = 4.45$ mm for that particular crack shape. The remaining ligament length $l_{lig} = 11.43$ mm, which can be expressed as $l_{lig}/2b = 0.55$. Prof. Schijve suggested first a comparison with Westergaard, but these values for straight through cracks as I can see it, are too low compared with the 3D straight through crack solutions I produced. And that means that straight through cracks would have a better crack growth behavior. I would leave it as it is. Comparing with my solutions is the most direct comparison. I made comparisons in my thesis of a part-through through crack vs. a straight through crack. You can reference that if you feel that comparison is worthy of mention. The more profound influence of crack interaction for straight through cracks can be found when the remaining l_{lig} is half the rivet pitch (50%) or $l_{lig}/2b = 0.50$. The more noticeable effect of

crack interaction for curved cracks $a/t = 1.05$ and $r/t = 1.0$ will be at $a/c_1 = 0.4$, that means $l_{lig} = 6.985 \text{ mm}$ $l_{lig}/2b = 0.272$.

The aforementioned comparison between straight through cracks and part-elliptical cracks shows clearly the influence of the crack shape. For $a/c_1 = 0.6$, constant a/t , and increasing r/t , crack interaction decreases. This is counter-intuitive, the crack interaction being dependent on the r/t ratio, but when increasing the r/t ratio the crack length c_1 decreases. The hole radius is kept constant throughout this investigation, this means that when r/t increases, t decreases. Since a/t is constant, a must decrease to keep the a/t ratio constant. Furthermore to keep a/c_1 constant, c_1 must decrease.

Oblique part-elliptical cracks for large a/c_1 ratios, large a/t and increasing r/t are similar to straight through cracks, the oblique part-elliptical shape is almost a straight through crack. These large values of a/c_1 show similar crack interaction effects as straight through cracks.

For $a/c_1 = 1.0$, $r/t = 2.0$ and $a/t = 3.0$, an almost straight crack front can be seen. For these values, the crack length on the faying surface c_1 is equal to 3.81 ($l_{lig} = 50\%$ of the rivet pitch). In this case the same interaction effect can be found as can be found for straight cracks.

Oblique part-elliptical through cracks with an almost straight crack front, deep cracks, show the same crack growth as straight cracks do (crack interaction around 50% of rivet pitch). Oblique part-elliptical through cracks with a more curved crack front, shallow cracks, show less crack interaction effect than the straight cracks do. Shallow cracks grow slower towards another (crack interaction will show a significant effect for a remaining l_{lig} ranging from 50% or less of the rivet pitch).

5 RESULTS

Fatigue crack growth results obtained with an optical traveling microscope and from SEM fractographic analysis are discussed below. The results indicate interactions between the cracks from the two holes. Predictions on these observations are based on the new K-solutions. Several corrections appear to be necessary.

5.1 EXPERIMENTALLY DETERMINED CRACK INTERACTION

Since the cracks have a curved front, the interaction effect on the crack growth will be first noticed by the crack length on the faying surface. The crack growth data show that when the crack length c_1 reaches a value between 5 and 6 millimeters, an increase in crack growth rate is observed. The corresponding ligament length (l_{lig}) is between 10.3 mm and 8.3 mm, or 40% and 32% of the hole pitch respectively. **Figure 15** then shows an increase of the crack growth rate for the inside cracks. The inside cracks have grown with the same growth rate as the outside cracks until this interaction between the cracks becomes noticeable. This behavior was found for all specimens with a starter notch, independent of the remote stress. Specimens tested without starter notches showed some non-uniform crack growth caused by a difference of the crack initiation time (for $k = 1.0$ initiation around 40 kcycles [2]). This non-uniform crack growth caused some cracks to slow down for some time, and others to grow to a larger size. Crack growth of this nature makes accurate fatigue life prediction more difficult.

Crack growth on the free surface of the specimen (crack length c_2 , Fig. 13) is dependent on the crack shape, which in turn is dependent on the bending. For a remote stress of 100 MPa and $k = 1$, the crack shape, $a/c_1 \approx 0.45$ at the point of crack interaction with $c_1 \approx 6.0 \text{ mm}$ and $c_2 \approx 4.0 \text{ mm}$.

Crack growth for a remote stress of 50 MPa, $k = 1.66$, starting with an initial crack shape $a/c_1 = 0.60$ ($c_1 = 3.5$ mm, $c_2 = 1.07$ mm), indicated crack interaction starting at $c_1 \approx 6.5$ mm and $c_2 \approx 3.0$ mm. Furthermore, a test done with a different pre-crack shape (initial shape $a/c_1 = 2.0$, $c_1 = 1.05$ mm, $c_2 = 0.32$ mm) showed that the shape of the growing crack is independent of the initial pre-crack. The crack shape is dependent on k only, which is a function of the applied remote stress. Looking closely at a/c_1 for $k = 1$ and $k = 1.66$, a difference is found. In general, crack interaction for $k = 1.0$ starts at $a/c_1 \approx 0.45$ and for $k = 1.66$ at $a/c_1 \approx 0.35$. This indicates that a different crack shape is found for the two k -values and that the crack shape for $k = 1.66$ is more shallow than for $k = 1.0$; automatically resulting in a delayed crack interaction for $k = 1.66$. The crack shapes for $k = 1.0$ and $k = 1.66$ at the region of crack interaction can not be seen as cracks with a steep crack front, i.e. cracks with a large a/c_1 value. The observed crack shapes should be considered to be shallow cracks characterized by a rather small a/c_1 value.

5.2 CRACK GROWTH PREDICTIONS

The crack growth predictions make use of the newly calculated stress intensity factors. These K solutions are contained in input files, one file with the β solutions for each of the three load cases, tension, bending, and pin loading. The β -values are read by the life prediction program. The tabulated β 's in [ref \[2,10\]](#) have been modified to represent the geometry of the specimens tested in this investigation. The corrections are explained in this section. The crack growth predictions are made with the United States Air Force AFGROW computer program developed by Harter [\[12\]](#). This program can handle a wide variety of fatigue problems, also the case for oblique through cracks. Currently, AFGROW is the only publicly available crack growth code with this capability. The calculated stress intensity factor solutions of the present study are for part-elliptical through cracks emanating from an infinite row of holes. These factors, available in [ref \[2,10\]](#), should thus be corrected for the finite width geometry of the specimen.

- A finite width correction is needed to account for the influence of the edges of the specimen on the eccentric holes. [Eqn. \(4\)](#) is used to correct the calculated solutions for cracks growing from eccentric holes in the specimen [\[11\]](#). [Eqn.\(3\)](#) is the corrected β .

$$\beta_i = \beta_{T,B} \cdot f_i \quad (3)$$

$$f_i = \sqrt{\sec \frac{\pi}{6} \left(\frac{1.45 \cdot \bar{c}}{d} + \frac{1.55 \cdot \bar{c}}{W - d} \right)} \quad (4)$$

where:

β_T, β_B	= Tension and bending correction factor
f_i	= Correction factor for inside crack tips growing towards center, for eccentric cracks
W	= the width of the sheet
d	= distance from the center of the crack to the closer edge of the sheet
\bar{c}	= crack length c_1 plus hole radius, for $\bar{c} > R$

- The crack interaction effect for the inside cracks is already included in the FEA results
- How to handle the effect of the constrained edges in the finite element analysis? The effect of the constrained edges, which introduces a σ_{xx} at $x = b$, appears to have a negligible influence on the crack interaction. This can be concluded from [Figure 14](#), which shows the uniaxial stress solution from [ref \[1\]](#) and the present study.

With corrected β 's in this form, the stress intensity factors for the combined tension and bending specimens can be calculated.

The crack growth prediction is made for through cracks with a known initial flaw shape. Two different initial flaw shapes are investigated. Before β_i can be used in AFGROW another correction needs to be made. AFGROW will add the Newman/Raju [\[13\]](#) finite width correction to β_i . This correction is superfluous in view of the other corrections made, and it should be removed. This is done by dividing β_i with this finite width correction.

Crack growth predictions of oblique through cracks are affected by the initial flaw shape; thus predictions are only made for specimens with EDM-notches. The tests to which the prediction will be compared have EDM manufactured initial flaw shapes, $a/c_1 = 1.0$ and $a/c_1 = 0.6$.

Before predictions are presented, the crack growth at the free surface shall be explained. It was already shown that the stress at the free surface is equal to zero for $k = 1.0$ and equal to -33 MPa for $k = 1.66$. The initial stress intensity factor for $k = 1.66$ is negative. This means that no crack growth will be possible at the free surface until $K > 0$. Since the specimen is experiencing an axial and a bending stress, the stress intensity factor is a superposition of K_{ten} and K_{ben} :

$$K = (\beta_{ten} \sigma_{ten} + \beta_{ben} \sigma_{ben}) \cdot \sqrt{\pi c_1} \quad (7)$$

For $k = 1.66$:

$$\begin{aligned} \sigma_{ten} &= 50 \text{ MPa} \\ \sigma_{ben} &= 83 \text{ MPa} \end{aligned}$$

The calculated β 's can be found in [ref \[3,10\]](#). The condition for crack growth is:

$$\begin{aligned} &(\beta_{ten} \sigma_{ten} + \beta_{ben} \sigma_{ben}) > 0 \\ \text{or } &|\beta_{ten} \sigma_{ten}| > |\beta_{ben} \sigma_{ben}| \end{aligned}$$

If the above mentioned condition is satisfied, crack growth will occur at the free surface.

[Figure 16](#) and [Figure 17](#) show the crack growth predictions for the inside cracks compared to the experimental results. Results for two different starter notches are shown ($a/c_1 = 0.6$ and $a/c_1 = 1.0$). For $a/c_1 = 0.6$ the crack growth curve for two different remote stress levels ($k = 1.0$ and $k = 1.66$) will be shown. For $a/c_1 = 1.0$ the crack growth curve is shown for only one stress level ($k = 1.66$). The predictions shown in [Figures 16, 17 and 18](#), are started using the first measured crack length after crack growth was observed.

The free surface is usually experiencing compressive stress through most or all of the load cycle, and thus crack opening is prohibited which hampers crack length measurement. The crack length measurements for small cracks at the free surface should be used with some reservations. This becomes more obvious when looking at Figure 18, although the prediction follows a similar trend as the experimental data. The crack growth at the free surface is characterized by a low crack growth rate until the cracks link up at the faying surface. After link up occurs the crack growth rate at the free surface increases dramatically for both, $k = 1.0$ and $k = 1.66$. For $k = 1.0$, link up at the free surface follows almost immediately; for $k = 1.66$ it occurs within a few kilocycles.

6 CONCLUSIONS

Fatigue tests were performed on combined tension and bending specimens with two holes. These specimens were designed to capture the influence of the secondary bending and interaction of cracks growing from different holes towards one another.

Visual crack measurements on the faying surface could be made with good accuracy, and similar crack growth curves were found for visual and SEM measurements. Crack length measurements for $k = 1.66$ ($k = \sigma_{\text{BENDING}}/\sigma_{\text{TENSION}}$) on the free surface caused some unexpected problems, due to the compressive stress for small crack lengths on the free surface.

SEM measurements could only be done for small cracks. Due to the tortuous fracture surface, no marker bands were found for larger cracks, both for $k = 1.0$ and $k = 1.66$. In the latter case, another problem occurred at the free surface due to the compressive stresses. The marker bands were then destroyed or masked by fretting debris. The marker load cycles did not affect the fatigue life of the specimens.

Stress intensity factors were calculated for various oblique crack front shapes with a/c_1 ratios of 0.3 to 10.0; a/t ratios of 1.05 to 5.0; and r/t ratios of 1.0 to 2.5. Calculations were made for hole edge cracks under remote tension, remote bending and pin loading (cosine² load distribution).

The analytical investigation showed that oblique part-elliptical through cracks with an almost straight crack front and a non-shallow shape show the same crack growth as straight cracks. Interaction between such crack occurred if cracks covered about 50% of hole pitch. Oblique part-elliptical through cracks with a more curved crack front and a shallow character show less crack interaction effects than the straight cracks. The shallow cracks are growing slower towards one another. Crack interactions become significant if the remaining ligament is smaller than 50% of the rivet pitch).

For the crack growth predictions a number of corrections were needed, first a set of corrections to correct the β 's for the differences in geometry between the test specimen and the finite element geometry. A second correction was needed to correct for the applied finite width correction in AFGROW. The predictions done with these corrections show a good agreement with the experimental crack growth results.

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