A Simplified Approach for Stress Analysis of Mechanically Fastened Joints

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The neutral line model is a simple analytical tool to calculate the stresses in each structural element in a lap-splice or butt joint. The method is verified by comparing analytical and experimental results. The model accounts for the eccentricities inherent in a multi-layer joint and uses advanced beam theory to derive the analytical solution for the in-plane and out-of-plane displacements that in turn can easily be converted to tension and bending stresses. For bending susceptible joints, thin sheet lap-splice joints, the influence of fastener rotation must be considered. In addition, the influence of longitudinal stiffeners is evaluated. Strain gage data was collected close to the most critical fastener row for the entire load range. Good correlation is observed between the analytical and experimental results for all joints tested. This analytical method provides a good first approximation of the stresses at the most critical fastener row thereby eliminating the need for labor intensive and time consuming finite element calculations.

1 Introduction

In conducting strength and damage tolerance analyses of mechanically fastened joints, knowing the stresses at the most critical fastener row is essential. This is the area most susceptible to fatigue crack nucleation and crack growth. For common riveted, longitudinal lap-splice joints, the typical loads are: tension introduced by the pressurization of the fuselage, secondary bending caused by the eccentricities of the joint and pin loading due to the load transfer through the fasteners. The objective of this research is to show that a good first approximation of the tension and secondary bending stresses are possible without labor intensive and time-consuming finite element calculations.

The theory used to derive the analytical model is based on advanced beam theory. By solving the resulting system of equations, the in-plane and out-of-plane displacements for all parts of the joint (beam parts) are obtained.[1] A joint is divided into several different beam parts, with the sheets between the fasteners behaving as beam elements with equal displacements and strains at the faying surfaces. The structural response of the joint is determined at the joint neutral line; thus, the sheets between the fasteners act as single beams.

In order to validate the neutral line model, four different joints are tested to obtain applied load vs. strain data. Two thin sheet lap-splice joints and two butt joints are tested, each instrumented with a number of strain gages at and near the critical fastener row.

2 Advanced Beam Theory

Extending Schijve's[1,2] neutral line model to more complicated structures has been successfully demonstrated in the past.[3] Analytical models for each of the four joint types are derived to give the global structural response of the joint to applied remote tensile loading. The models are verified by comparing the analytical to experimental results.

2.1 Neutral Line Model

Whether the structure is simple or complex, the procedure to develop a neutral line model remains the same. The model is one-dimensional in that the structural response is confined to the neutral line. A one-dimensional lap-splice joint with misalignment and clamped at the sheet ends is shown in Figure 1. The deflected shape of the lap-splice joint is shown in Figure 2. The stresses in the area of interest are then easily obtained as shown below, starting with static equilibrium of the specimen:

$$\sum M_{\text{Point A}} = 0$$

$$M_a - M_b - Pa - D_b l_{tot} = 0 \qquad (1)$$

with $\quad l_{tot} = L_1 + L_2 + L_3 + L_4$

Equilibrium for each separate "beam" part can be written as follows

$$(M_x)_i = M_a + Pw_i - D_a\left(\sum_1^i L_{i-1} + x_i\right) = (EI)_i\left(\frac{d^2w}{dx_2}\right)_i$$

Or in a more useful form

$$\left(\frac{d^2w}{dx^2}\right)_i - \alpha_i^2 w_i = \alpha_i^2\left(\frac{M_a}{P} - \frac{D_a}{P}\left(\sum_1^i L_{i-1} + x_i\right)\right) \qquad (2)$$

with $\quad \alpha_i^2 = \dfrac{P}{(EI)_i}$

and
- P = Remote applied axial load
- M_a = Moment due to clamping
- D_a = Reaction Force due to clamping
- E_i = Modulus of Elasticity for the i^{th} part of the beam
- I_i = Moment of Inertia for the i^{th} part of the beam

Solution of Eqn. (2) results in a second order linear equation for the i^{th} "beam" part

$$w_i = A_i \sinh(\alpha_i x_i) + B_i \cosh(\alpha_i x_i) + \left(\frac{D_a}{P} \left(\sum_1^i L_{i-1} + x_i \right) - \frac{M_a}{P} \right) \quad (3)$$

Boundary conditions for rivet positions

$$w_j + e_j = w_{j+1} \quad \text{and} \quad \left(\frac{dw}{dx}\right)_j + \beta_j = \left(\frac{dw}{dx}\right)_{j+1} \quad (4)$$

Special attention should paid to the e_j and β_j, being positive or negative! Figure 1 shows that e_1, the eccentricity jump from part 1 to part 2, is negative. Since the effect of the load transfer is not taken into account yet, $e_2 = 0$ and finally e_3 will have the same sign as e_1. As for the effect of rivet rotation β_j, this is more complicated. At the first fastener row, see Figure 3, the boundary condition is

$$\left(\frac{dw}{dx}\right)_1 + \beta_1 = \left(\frac{dw}{dx}\right)_2$$

According to reference [1], a relative low stress level can already cause plastic deformation. This will cause a locally small permanent (plastic) bending of the sheets. To get an idea of the effects of rivet rotation on the stress calculations, it is assumed that the permanent bending is concentrated at the first/critical fastener row. This changes the boundary condition for the middle fastener row to

$$\left(\frac{dw}{dx}\right)_2 = \left(\frac{dw}{dx}\right)_3$$

thus, no β effect is taken into account. Finally at the third fastener row, the boundary condition will be

$$\left(\frac{dw}{dx}\right)_3 + \beta_3 = \left(\frac{dw}{dx}\right)_4$$

<u>Part I</u>

The solution of Eqn. (3), yields

$$w_1 = A_1 \sinh(\alpha_1 x_1) + B_1 \cosh(\alpha_1 x_1) + \left(\frac{D_a}{P} \cdot x_1 - \frac{M_a}{P} \right) \quad (5)$$

The boundary conditions for the clamped situation at $x_1 = 0$ yield, displacement $w_1 = 0$ and $\left(\frac{dw}{dx}\right)_{1,x_1=0} = 0$. Substitution in Eqn. (5) and the first derivative of Eqn. (5) results in a solution for A_1 and B_1

$$A_1 = -\frac{D_a}{\alpha_1 P} \quad (6)$$

$$B_1 = \frac{M_a}{P} \quad (7)$$

Part II

$$w_2 = A_2 \sinh(\alpha_2 x_2) + B_2 \cosh(\alpha_2 x_2) + \left(\frac{D_a}{P}(L_1 + x_2) - \frac{M_a}{P} \right) \tag{8}$$

Using Eqn. (4)

$$w(x_2 = 0)_2 = w(x_1 = L_1) + e_1$$

$$B_2 = A_1 \sinh(\alpha_1 L_1) + B_1 \cosh(\alpha_1 L_1) + e_1 \tag{9}$$

$$\left(\frac{dw}{dx} \right)_{2, x_2 = 0} = \left(\frac{dw}{dx} \right)_{1, x_1 = L_1} + \beta_1$$

$$A_2 \alpha_2 = A_1 \alpha_1 \cosh(\alpha_1 L_1) + B_1 \alpha_1 \sinh(\alpha_1 L_1) + \beta_1 \tag{10}$$

Part III

$$w_3 = A_3 \sinh(\alpha_3 x_3) + B_3 \cosh(\alpha_3 x_3) + \left(\frac{D_a}{P}(L_1 + L_2 + x_3) - \frac{M_a}{P} \right) \tag{11}$$

Using the same principle as was done for part 2, results in

$$w(x_3 = 0)_3 = w(x_2 = L_2) + e_2$$

$$B_3 = A_2 \sinh(\alpha_2 L_2) + B_2 \cosh(\alpha_2 L_2) + e_2 \tag{12}$$

$$\left(\frac{dw}{dx} \right)_{3, x_3 = 0} = \left(\frac{dw}{dx} \right)_{2,\ x_2 = L_2}$$

$$A_3 \alpha_3 = A_2 \alpha_2 \cosh(\alpha_2 L_2) + B_2 \alpha_2 \sinh(\alpha_2 L_2) \tag{13}$$

Part IV

$$w_4 = A_4 \sinh(\alpha_4 x_4) + B_4 \cosh(\alpha_4 x_4) + \left(\frac{D_a}{P}(L_1 + L_2 + L_3 + x_4) - \frac{M_a}{P} \right) \tag{14}$$

Using Eqn. (4):

$$w(x_4 = 0)_4 = w(x_3 = L_3) + e_3$$

$$B_4 = A_3 \sinh(\alpha_3 L_3) + B_3 \cosh(\alpha_3 L_3) + e_3 \tag{15}$$

$$\left(\frac{dw}{dx} \right)_{4, x_4 = 0} = \left(\frac{dw}{dx} \right)_{3,\ x_3 = L_3} + \beta_3$$

$$A_4 \alpha_4 = A_3 \alpha_3 \cosh(\alpha_3 L_3) + B_3 \alpha_3 \sinh(\alpha_3 L_3) + \beta_3 \tag{16}$$

Similar to $x_1 = 0$, at $x_4 = L_4$ the displacement *w* is equal to the misalignment ‘a’ and no rotation is allowed due to the clamping of the specimen, yielding the last two equations

$$A_4 \sinh(\alpha_4 L_4) + B_4 \cosh(\alpha_4 L_4) + \left(\frac{D_a}{P} \cdot L_{tot} - \frac{M_a}{P} \right) = a \quad (17)$$

$$A_4 \alpha_4 \cosh(\alpha_4 L_4) + B_4 \alpha_4 \sinh(\alpha_4 L_4) + \frac{D_a}{P} = 0 \quad (18)$$

Table 1 shows the system of 10 equations that need to be solved for 10 unknowns, A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4, D_a and M_a. Solving these equations provides the displacements of the neutral line for all x, and offers the opportunity to calculate the bending moment at the location of interest, the first fastener row

$$M_1 = M_a + P w_{1, x_1 = L_1} - D_a (L_1) \quad (19)$$

The equations shown in Table 1 can be simplified if desired without significantly affecting the accuracy of the solution. The first step towards a more simple system of equations is that, according to Schijve, the influence of fixed or clamped boundary conditions on the stresses in a joint can be neglected for large values of $\alpha_1 L_1$, which is common for thin sheet joints.[1] This results in a reduction in the number of unknowns, D_a and M_a are no longer required, and a reduction in the number of equations needed, Eqn. (6) and Eqn. (18). Since the fastener head, manufactured and driven, are not modelled explicitly, a point of symmetry exists at the second fastener when $t_1 = t_2$ and $E_1 = E_2$. Table 2, shows the reduced system of equations.

The degree or severity of bending with respect to tensile loading is commonly represented by the bending factor, k the ratio of the nominal bending to tensile stresses. At the first fastener row, the k is

$$k = \frac{\sigma_b}{\sigma_t} = \frac{M_1}{\frac{1}{6} W t_1^2} \Bigg/ \frac{P}{W t_1} \quad (20)$$

Where

W = joint width.
t_1 = thickness of the joint at the first fastener row

Since the model is one-dimensional, the bending factor is not a function of the joint width. Although for finite width lap-splice joints, Muller[4] and Fawaz[5] both measured larger k's at the specimen edges due to the finite width effect. The bending stress through the thickness at the first fastener row is then

$$\sigma_b(y) = \frac{M_1 y}{I_1} \quad (21)$$

Where

y = distance from the neutral line to point of interest in the thickness direction.

3 Experiments

Four different specimens representative of transport aircraft fuselage skin joints were used to obtain load vs. strain data. See Figure 4 for general dimensions of the test specimens. All specimens are manufactured from 2024-T3 clad aluminum and subjected to a remote cyclical tension load. The specimens were instrumented with a number of strain gages at the critical rivet row and approximately one inch away from the critical rivet row. All comparisons between the predicted and measured strain data are at the latter location. A complete discussion of the experimental investigation can be found in reference [5]. Figure 5 shows the four different joint configurations.

JOINT I, is a four-rivet row lap-splice joint with extra doublers and a longitudinal stiffener; the latter being attached with only one fastener row. Strain gages for this joint were attached one inch to the right of fastener row A.

JOINT II is a three-rivet lap-splice joint with a longitudinal stiffener attached with only one fastener row at the position of the middle fastener row. Same place for the strain gages has been chosen, namely one inch to the right of row A.

Implementing the stiffeners from the first two lap-splice joints required a different approach, since those longitudinal stiffeners are attached via only one fastener. This is resolved using the relation that some types of fasteners allow for a clamping effect around the fastener. The applied pressure of the fastener head on the sheets causes clamping. Fastener installation results in a variety of different stress systems in the surrounding sheet material. Installing a solid rivet will cause a residual stress around the fastener hole and a clamping force as a result of the forming of the driven rivet head. Other fasteners used are “lock-bolts”, when using clearance fit, no residual stress will be present in the sheet material surrounding the installed fastener. Interference fit fasteners will introduce a residual stress, albeit less effective as can be seen for a solid rivet. The clamping force for the “lock-bolt” type fasteners is dependent on the torque force used to install the fastener nut.

JOINT III is a butt joint with an extra doubler and longitudinal stiffener. The longitudinal stiffener is attached to the joint with two fastener rows.

JOINT IV is a but-splice joint with an extra doubler. All joints were equipped with strain gages one inch to the right of row A.

4 Results and Discussion

The measured strains were converted to stress using one-dimensional Hooke’s law. The stress at the given location is the total normal stress, which has both tension and bending components. The individual stresses are decomposed using the following relation.

$$\sigma_t = \frac{\sigma_i + \sigma_j}{2} \qquad \sigma_b = \frac{\sigma_i - \sigma_j}{2} \tag{22}$$

Since the strain gages are mounted back-to-back on each side of the sheet, the i,j indices simply indicate whether the strain gage is on the front or back. Finally, the stresses from the test are compared to the calculated stress from the neutral line model. Figure 6 to 9 show the applied load vs. stress curves for each of the joint types. These figures clearly show the influence of the fastener rotation, which is discussed below.

JOINT I

As expected, the calculated axial stresses show good correlation to the tensile stress component, see Figure 6. The bending component is dependent on the rivet rotation and albeit minimal, the attached longitudinal stiffener. Since the stiffener is attached by means of a single fastener row, it does not significantly affect the joint flexibility. Including the longitudinal stiffener into the derivation of the neutral line model increases the complexity of the derivation without a noticeable increase in the accuracy of the calculated stresses.

As load is transferred from the one sheet to the next through the rivets, the rivets rotate, more so in the outer rivet rows. This rotation causes a small permanent bending β as was mentioned before. By including this rivet rotation, the correlation increases between the predicted and measured stresses. By not considering rivet rotation, the calculated bending stresses are always conservative, see Figure 6. Two calculations are made with the influence of fastener rotation due to the small permanent deformation at the beginning of the effective overlap, one calculation using $\beta = 1°$ and one with $\beta = 0.5°$. It can be seen that the accuracy of the calculated bending stresses increases with increasing fastener rotation. As mentioned before in the derivation of the differential equations, β is introduced at the critical fastener row. However for joint type I, this is slightly different because of the doublers which are added to allow a gradual load transfer between the two main sheets. Since the doublers have a small thickness, they don't transfer a similar amount of load as the main sheet. This directly affects the load transfer of the two middle rows, and thus these two fastener rows carry a larger part of the load. For this reason, β will be used for these fasteners rows also.

JOINT II

This lap-splice joint is simpler than the joint I since there are no doublers. For this joint configuration, the influence of the stiffener is also minimal for the same reasons stated above. The longitudinal stiffener is not attached to the joint to provide restraint to the secondary bending, but to increase the stiffness in longitudinal direction (joint width) to carry fuselage bending loads. Figure 7 shows the measured and calculated stresses; again the neutral line model accurately calculates the axial tensile stresses. The bending stresses are influenced by the fastener rotation, and was seen for joint I, the accuracy of the calculated stresses increase with increasing fastener rotation. For both joint types, the optimum results are obtained using a fastener rotation $\beta = 1°$, which is the same value used by Schijve.[2]

JOINT III

Accurate predictions of both the tensile and bending stresses are seen in Figure 8. Notice good correlation is found without considering rivet rotation. When the fastener rotation is implemented used in the prediction, the bending stresses decrease with increasing fastener rotation. Decreasing bending stresses implies the stresses become less positive or negative. The following question then arises, why do we get good correlation without rivet rotation for the butt joints. The butt joints used in this investigation have a somewhat thicker sheet material compared to the lap-splice joint. Due to the increased flexural rigidity of the butt splices, they don't experience significant plastic bending and thus rivet rotation is negligible.

JOINT IV

The comparison in Figure 9 shows that this butt joint behaves in a similar manner as joint III, with respect to the fastener rotation. As expected, the butt joints are in general less flexible than lap-splice joints.

5 Conclusion

From Figure 6 to 9, it is clear that our extension of the Schijve neutral line model provides an accurate and simple means to estimate the stresses at the most critical fastener row in a joint. For thin sheet lap-splice joints, which are more flexible, the differences in joint flexibility can be taken into account by introducing a fastener rotation, β. The fastener rotation provides a physically based means to account for the added flexibility found in lap-splice joints thereby allowing a more accurate calculation of the bending stress. This allows us to say that for flexible lap-splice joints an upper limit of $\beta = 1°$ can be taken. It is shown that the tensile and bending stresses in butt joints, which are less flexible due to increased sheet thickness, can be calculated without taking any fastener rotation into account.

6 References

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