Center for Aircraft Structural Life Extension

Providing Structural Integrity Technology to the Aerospace Community

Use of Bending Stress Intensity Factor Solutions for Wide, Thin Plates

Börje Andersson, BARE AB
Scott Fawaz, SAFE Inc.

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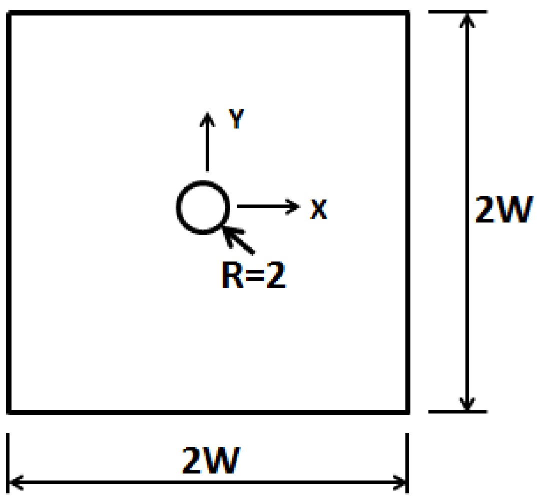
Outline

- Background
- Design & Relevance
- Error Assessment
- Options
- Conclusions

Background

- *K* solutions created in 2003
- Discretized domain



  - 144 FE in xy plane
  - Four layers in z-dir

- Cracking scenarios
  - Single & Double
- Geometry
  - $0.1 \leq a/c \leq 10.0$
  - $0.1 \leq a/t \leq 0.99$
  - $10 \geq R/t \geq 0.1$
  - $0.2 \leq t \leq 20$
  - $W/D = 100$
- Loading
  - Tension
  - Bending
  - Pin (bearing)

Errors in *K* Database

- Discovered in 2013
  - Numerical round-off
  - Nonlinear geometry effects
- Cracking scenarios
  - Single
  - Double
- Geometry
  - *R/t* > 3.0
- Loading:
  - Tension
  - Bending
  - Pin (bearing)
- Advanced Models
  - 2-point and multi-point

Loading/Tip	*a/t*	*R/t*
Bending/a	0.10	4,5,6,7,10 (all a/c)
Bending/a	0.50	4,5,6,7,10 (all a/c)
Bending/a	0.95	4,5,6,7,10 (all a/c)
Tension/a	0.10	7,8,9,10 (a/c very small)
Tension/a	0.50	7,10 (a/c very small)
Tension/a	0.95	7,8,9,10 (a/c, small mainly)
Bearing/a	0.10	8,9,10 (a/c very small)
Bearing/a	0.50	7,10 (a/c very small) 0.5, a/c very large
Bearing/a	0.95	7,8,9,10 (a/c, small mainly) 0.5, a/c very large
Bending/c	0.10	4,5,6,7,8,9,10
Bending/c	0.50	4,5,6,7,10
Bending/c	0.95	4,7,9,10
Tension/c	0.10	8,9,10 (a/c large)
Tension/c	0.50	7 (a/c =10)
Tension/c	0.95	7,8,9,10 (a/c large)
Bearing/c	0.10	8,9
Bearing/c	0.50	7,8,9
Bearing/c	0.95	7,8,9

Design & Relevance

- Min fastener spacing → 4D[1]
  - Implies *W/D* = 4
- Avoid *R/t* > 2.0[2:8-15]
- Joint Allowables
  - Ultimate Bearing Strength, Solid Rivets
    - 0.125 ≤ *R/t* ≤ 2.64[2:8-17]
  - Static Joint Strength (SJS), A-286/A-286
    - 0.58 ≤ *R/t* ≤ 3.125[2:8-17]
  - SJS, A-286 Blind Rivets, SS, Ti, AA Sheet
    - 1.0 ≤ *R/t* ≤ 7.81[2:8-39]
- SJS, Monel Rivets, SS Sheet
  - 0.78 ≤ *R/t* ≤ 6.25[2:8-40]
- SJS, Blind Monel Rivets, AA Sheet
  - 0.65 ≤ *R/t* ≤ 2.50[2:8-41]
- SJS, Blind AA Rivets, AA Sheet
  - 1.17 ≤ *R/t* ≤ 2.50[2:8-42]
- SJS, Shear Head Ti bolts, AA Sheet
  - 0.65 ≤ *R/t* ≤ 1.56[2:8-114]
- UBS, Threaded Fasteners
  - 0.078 ≤ *R/t* ≤ 2.43[2:8-131]
- SJS, Steel Hi-Lok, AA Sheet
  - 0.625 ≤ *R/t* ≤ 1.56[2:8-146]

[1]Niu, Airframe Stress Analysis and Sizing, 2nd Ed., p.277.
[2]MMPDS-04.

Error Control - 2003

- Used a large domain to avoid finite height/width effects
  - Rely on AFGROW finite width correction
- Through the thickness mesh design
  - Graded (aka "*hp*") – higher accuracy for small R/t
  - Uniform – 4 equal layers for large R/t
- Polynomial order, $p = 6$

Error Analysis

- Solving the linear system of equations
  - $\{F\} = [K]\{u\}$
    - $\{F\}$ - generalized loads
    - $[K]$ - stiffness matrix
    - $\{u\}$ - N unknown displacements
- Error, e
  - $e = \frac{\sqrt{\sum_{i=1}^{i=N}(u_i - v_i)^2}}{\sqrt{\sum_{i=1}^{i=N} v_i^2}}$
    - v - approximation to u

hp Mesh Errors

- $p = 8$
- $W = 200$

R/t	Tension	Bending	Wedge
0.04	0.1E-13	0.1E-12	0.6E-13
0.1	0.9E-10	0.6E-08	0.3E-09
0.2	0.2E-9	0.3E-07	0.7E-09
0.5	0.1E-8	0.6E-6	0.4E-8
1.0	0.6E-8	0.3E-5	0.2E-7
2.0	0.2E-7	0.4E-4	0.7E-7
3.0	0.6E-7	0.2E-3	0.9E-7
4.0	0.1E-6	0.2E-3	0.3E-6
5.0	0.2E-6	0.5E-3	0.5E-6
6.0	0.3E-6	0.8E-3	0.5E-6
7.0	0.4E-6	0.1E-1	0.2E-5
8.0	0.4E-6	0.2E-2	0.2E-5
10.0	Singular	Singular	Singular

- Red highlights unacceptable errors
- For small *R/t* – errors are within machine precision
- For increasing *R/t* – errors increase quickly
  - Especially for bending
- Must use uniform mesh to minimize round-off errors

Uniform Mesh Errors-Tension

- Tension, *p* = 8
- *10 ≤ W ≤ 200, R/t*=10

Z	W=10	W=25	W=50	W=100	W=200
-0.200	3.3569	3.0519	3.0108	3.0005	2.9991
-0.195	3.3574	3.0524	3.0112	3.0010	2.9991
-0.190	3.3578	3.0528	3.0116	3.0014	2.9992
-0.180	3.3587	3.0536	3.0124	3.0022	2.9998
-0.160	3.3603	3.0549	3.0137	3.0035	3.0011
-0.140	3.3614	3.0559	3.0147	3.0045	3.0019
-0.120	3.3620	3.0565	3.0153	3.0050	3.0025
-0.100	**3.3622**	**3.0567**	**3.0155**	**3.0052**	**3.0027**
-0.080	3.3620	3.0565	3.0153	3.0050	3.0025
-0.060	3.3614	3.0559	3.0147	3.0045	3.0019
-0.040	3.3603	3.0549	3.0137	3.0035	3.0009
-0.020	3.3587	3.0536	3.0124	3.0022	2.9996
-0.010	3.3578	3.0528	3.0116	3.0014	2.9988
-0.050	3.3574	3.0524	3.0112	3.0010	2.9984
0.000	3.3569	3.0519	3.0108	3.0006	2.9980
Error →	0.8E-10	0.7E-9	0.2E-8	0.9E-8	0.5E-7

- Red highlights loss of symmetry
  - Problem is perfectly symmetric though
- Error is small
- Errors will not affect *K*

Uniform Mesh Errors-Tension

- Tension, $2 \leq p \leq 8$
- $W = 25$, $R/t=10$

Z	P=2	P=3	P=4	P=5	P=6	P=7	P=8
-0.200	3.0633	3.0621	3.0510	3.0521	3.0520	3.0519	3.0519
-0.195	3.0643	3.0623	3.0514	3.0525	3.0524	3.0524	3.0524
-0.190	3.0650	3.0626	3.0517	3.0530	3.0528	3.0528	3.0528
-0.180	3.0657	3.0631	3.0524	3.0538	3.0536	3.0536	3.0536
-0.160	3.0641	3.0642	3.0537	3.0551	3.0550	3.0549	3.0549
-0.140	3.0674	3.0662	3.0548	3.0561	3.0559	3.0559	3.0559
-0.120	3.0715	3.0669	3.0554	3.0567	3.0565	3.0565	3.0565
-0.100	**3.0766**	**3.0674**	**3.0556**	**3.0569**	**3.0567**	**3.0567**	**3.0567**
-0.080	3.0715	3.0669	3.0554	3.0567	3.0565	3.0565	3.0565
-0.060	3.0674	3.0662	3.0548	3.0561	3.0559	3.0559	3.0559
-0.040	3.0641	3.0642	3.0537	3.0551	3.0550	3.0549	3.0549
-0.020	3.0657	3.0631	3.0524	3.0538	3.0536	3.0536	3.0536
-0.010	3.0650	3.0626	3.0517	3.0530	3.0528	3.0528	3.0528
-0.050	3.0643	3.0623	3.0514	3.0525	3.0524	3.0524	3.0524
0.000	3.0633	3.0621	3.0510	3.0521	3.0520	3.0519	3.0519
Error →	0.4E-9	0.1E-9	0.1E-9	0.7E-10	0.1E-9	0.1E-9	0.6E-9

- Green highlights 5 digits accuracy
- Uniform mesh in the thickness direction gives accurate K solutions for $p > 5$

Uniform Mesh Errors-Bending

- Bending, *p* = 8
- *10 ≤ W ≤ 200, R/t*=10

Z	W=10	W=25	W=50	W=100	W=200
-0.200	-1.8776	-1.8190	-1.8002	-1.7641	-4.6761
-0.195	-1.7875	-1.7299	-1.7171	-1.6988	-3.2222
-0.190	-1.6976	-1.6411	-1.6319	-1.6227	-2.4329
-0.180	-1.5134	-1.4633	-1.4570	-1.4540	-1.8132
-0.160	-1.1411	-1.1034	-1.0981	-1.0967	-1.4001
-0.140	-0.7634	-0.7382	-0.7345	-0.7342	-0.6587
-0.120	-0.3825	-0.3698	-0.3681	-0.3686	-0.2863
-0.100	**0.00001**	**0.0001**	**-0.0001**	**-0.0002**	**-0.002**
-0.080	0.3825	0.3698	0.3681	0.3679	0.3723
-0.060	0.7634	0.7382	0.7346	0.7338	0.8052
-0.040	1.1411	1.1034	1.0981	1.0970	1.2115
-0.020	1.5134	1.4636	1.4567	1.4549	1.5920
-0.010	1.6976	1.6408	1.6329	1.6305	1.7781
-0.050	1.7874	1.7284	1.7201	1.7173	1.8667
0.000	1.8772	1.8153	1.8067	1.8032	1.9488
Error →	3.5E-8	0.3E-6	0.2E-5	0.2E-4	0.3E-3

- Red highlights loss of unsymmetric properties
- Errors increase with *W*
  - Errors significant for W=100
- For *W* > 50, *R/t*=10, *K* cannot be calculated accurately

Uniform Mesh Errors-Bending

- Bending, $2 \leq p \leq 8$
- $W = 25$, $R/t = 10$

Z	P=2	P=3	P=4	P=5	P=6	P=7	P=8
-0.200	-2.1781	-2.1625	-1.8152	-1.8148	-1.8151	-1.8147	-1.8190
-0.195	-2.0580	-2.0439	-1.7273	-1.7280	-1.7281	-1.7280	-1.7299
-0.190	-1.9397	-1.9283	-1.6394	-1.6406	-1.6406	-1.6406	-1.6411
-0.180	-1.7085	-1.7048	-1.4629	-1.4639	-1.4637	-1.4637	-1.4633
-0.160	-1.2674	-1.2747	-1.1038	-1.1036	-1.1035	-1.1033	-1.1034
-0.140	-0.8324	-0.8474	-0.7371	-0.7380	-0.7381	-0.7382	-0.7382
-0.120	-0.4125	-0.4215	-0.3698	-0.3699	-0.3698	-0.3698	-0.3698
-0.100	**0.001**	**0.001**	**-0.0001**	**0.00003**	**0.00003**	**0.00001**	**0.00001**
-0.080	0.4125	0.4215	0.3698	0.3699	0.3698	0.3698	0.3698
-0.060	0.8324	0.8474	0.7371	0.7380	0.7381	0.7382	0.7382
-0.040	1.2674	1.2747	1.1038	1.1036	1.1035	1.1033	1.1034
-0.020	1.7085	1.7048	1.4629	1.4639	1.4637	1.4637	1.4636
-0.010	1.9397	1.9284	1.6395	1.6406	1.6406	1.6407	1.6408
-0.050	2.0580	2.0439	1.7273	1.4639	1.7282	1.7282	1.7284
0.000	2.1781	2.1625	1.8152	1.6406	1.8152	1.8152	1.8153
Error →	**0.2E-7**	**0.2E-6**	**0.9E-7**	**0.2E-7**	**0.6E-7**	**0.5E-7**	**0.2E-6**

- Red highlights loss of unsymmetric properties
- Errors increase with *p*
- For $p > 6$, *K* cannot be calculated accurately
- 2003 solutions used $p = 6$

Error Summary

- Large errors in *K* when
  - *R/t* is large (*R/t* > 3.0)
  - Plate is wide
  - Linear analysis - example
    - σ_B = 100 MPa, *p* = 8
    - *R/t* = 8
    - *E* = 71 GPa
    - ν = 0.3
    - Results
      - *e* = 2E-3
      - *w*(0,200) = -800t
      - *u*(200,200) = 192t

Error Mitigation

- Use smaller *W* for large *R/t*
  - *W* = 400 / *R/t*
    - Would have to reanalyze everything except *R/t* = 2
- Add additional constraints for bending cases
  - At the surface X=0, X-displacements are zero due to symmetry
  - At the surface Y=0, Y-displacements are zero due to symmetry
  - At the surface Z=-t/2, X- and Y-displacements are zero due to anti-symmetry for the bending loading (only)
  - Adding the above decreased the round-off error by 2X for *R/t*=10 but 2X is not enough
- Implement a new thin-shell FEA formulation
  - Method available since 1992, but no resources available
- Consider geometric nonlinear effects

Options

- Table shows example of geometric nonlinearity parameter, $\rho = {}^{\sigma_{B_{NL}}}/_{\sigma_B}$ from nonlinear beam theory
- σ_T = 100MPa, $\sigma_B \neq 0$, E = 70 GPa

R/t	H=5	H=10	H=20	H=30	H=50	H=100	H=200
0.1	0.9995	0.9979	0.9916	0.9813	0.9494	0.8205	0.5074
0.2	0.9979	0.9916	0.9671	0.9285	0.8205	0.5074	0.1477
0.3	[illegible]	[illegible]	0.9285	0.8503	0.6604	0.2789	0.0405
0.4	0.9916	0.9671	0.8785	0.7576	0.5074	0.1477	0.0110
0.5	0.9869	0.9494	0.8205	0.6604	0.3791	0.0774	0.0030
0.6	0.9813	0.9285	[illegible]	[illegible]	0.2789	0.0405	0.0008
0.8	0.9671	0.8785	0.6284	0.4025	0.1477	0.0110	0.0001
1.0	0.9494	0.8205	0.5074	0.2789	0.0774	0.0030	0.0000
2.0	0.8205	0.5074	0.1477	0.0405	0.0030	0.0000	0.0000
3.0	0.6604	0.2789	0.0405	0.0058	0.0001	0.0000	0.0000
5.0	0.3791	0.0774	0.0030	0.0001	0.0000	0.0000	0.0000
10.0	0.0774	0.0030	0.0000	0.0000	0.0000	0.0000	0.0000

Option 2a Option 2b Option 1

Large R/t, H → Large error in K's using combined σ_T and σ_B from linear theory

Option 1

- *The linear and nonlinear solution differ dramatically*
- *Effects of the bending moment on the stresses near the hole are almost not visible in the nonlinear solution*
    - o *Stresses near the hole are almost symmetrical with respect to the mid-plane despite the large applied bending stresses*
- *The stresses in the nonlinear solution are practically the same as for the linear solution without the bending stresses*
- *The round-off errors in the nonlinear solution are much smaller than in the linear solution due to the geometrically nonlinear effects which kinematically stiffens the plate by mainly preventing out of plane displacements*
- *For thin, large plates (large W, R/t) the effects of bending on stress intensity factors can be neglected!*

$\sigma_Y(R,0,Z)$

Z	Linear Tension & Bending	Nonlinear Tension & Bending	Linear Pure Tension
-0.400	208.60	300.24	299.04
-0.390	233.79	300.40	299.21
-0.380	245.56	300.56	299.37
-0.360	254.59	300.85	299.66
-0.320	265.38	301.32	300.14
-0.280	277.47	301.65	300.49
-0.240	289.63	301.84	300.69
-0.200	**301.17**	**301.89**	**300.76**
-0.160	312.13	301.81	300.69
-0.120	323.38	301.59	300.49
-0.080	334.39	301.23	300.14
-0.040	344.86	300.73	299.66
-0.020	349.86	300.42	299.37
-0.010	352.30	300.26	299.21
0.000	354.69	300.08	299.04
Error →	**0.4E-5**	**0.7E-8**	**0.9E-8**

Options

- Table shows example of geometric nonlinearity parameter, $\rho = {}^{\sigma_{B_{NL}}}/_{\sigma_B}$ from nonlinear beam theory
- σ_T = 100MPa, $\sigma_B \neq 0$, E = 70 GPa

R/t	H=5	H=10	H=20	H=30	H=50	H=100	H=200
0.1	0.9995	0.9979	0.9916	0.9813	0.9494	0.8205	0.5074
0.2	0.9979	0.9916	0.9671	0.9285	0.8205	0.5074	0.1477
0.3	0.[illegible]	[illegible]13	0.9285	0.8503	0.6604	0.2789	0.0405
0.4	0.9916	0.9671	0.8785	0.7576	0.5074	0.1477	0.0110
0.5	0.9869	0.9494	0.8205	0.6604	0.3791	0.0774	0.0030
0.6	0.9813	0.9285	0.757[illegible]	[illegible]	0.2789	0.0405	0.0008
0.8	0.9671	0.8785	0.6284	0.4025	0.1477	0.0110	0.0001
1.0	0.9494	0.8205	0.5074	0.2789	0.0774	0.0030	0.0000
2.0	0.8205	0.5074	0.1477	0.0405	0.0030	0.0000	0.0000
3.0	0.6604	0.2789	0.0405	0.0058	0.[illegible]	[illegible]0	0.0000
5.0	0.3791	0.0774	0.0030	0.0001	0.[illegible]	[illegible]0	0.0000
10.0	0.0774	0.0030	0.0000	0.0000	0.0000	0.0000	0.0000

Option 2a — Option 2b — Option 1

Option 2a

- Accept the ≤ 5% error due to the geometric nonlinearity and use linear superposition as always
  - $K_{I_{total}} = K_{I_{Tension}} + K_{I_{Bending}}$
  - $K_I = \sigma_{Tension}\sqrt{\pi a}\beta_{Tension}\left(\frac{a}{c}, \frac{a}{t}, \frac{R}{t}, \frac{W}{D}, \varphi\right) +$

    $\sigma_{Bending}\sqrt{\pi a}\beta_{Bending}\left(\frac{a}{c}, \frac{a}{t}, \frac{R}{t}, \frac{W}{D}, \varphi\right)$
  - $\beta_{Tension}, \beta_{Bending}$ from AFGROW

Options

- Table shows example of geometric nonlinearity parameter, $\rho = {}^{\sigma_{B_{NL}}}/_{\sigma_B}$ from nonlinear beam theory
- σ_T = 100MPa, $\sigma_B \neq 0$, E = 70 GPa

R/t	H=5	H=10	H=20	H=30	H=50	H=100	H=200
0.1	0.9995	0.9979	0.9916	0.9813	0.9494	0.8205	0.5074
0.2	0.9979	0.9916	0.9671	0.9285	0.8205	0.5074	0.1477
0.3	[illegible]	[illegible]	0.9285	0.8503	0.6604	0.2789	0.0405
0.4	0.9916	0.9671	0.8785	0.7576	0.5074	0.1477	0.0110
0.5	0.9869	0.9494	0.8205	0.6604	0.3791	0.0774	0.0030
0.6	0.9813	0.9285	[illegible]	[illegible]	0.2789	0.0405	0.0008
0.8	0.9671	0.8785	0.6284	0.4025	0.1477	0.0110	0.0001
1.0	0.9494	0.8205	0.5074	0.2789	0.0774	0.0030	0.0000
2.0	0.8205	0.5074	0.1477	0.0405	0.0030	0.0000	0.0000
3.0	0.6604	0.2789	0.0405	0.0058	[illegible]	[illegible]	0.0000
5.0	0.3791	0.0774	0.0030	0.0001	[illegible]	[illegible]	0.0000
10.0	0.0774	0.0030	0.0000	0.0000	0.0000	0.0000	0.0000

Option 2a

Option 2b

Option 1

Option 2b

- $K_I = \sigma\sqrt{\pi a}\beta = (\sigma_{ten}\beta_{ten} + \sigma_{ben,NL}\beta_{ben})\sqrt{\pi a}$
  - σ_{ten} - user input
  - β_{ten} - from AFGROW
  - $\sigma_{ben,NL} = \rho\sigma_{ben}$
    - σ_{ben} - user input
    - ρ - from geometric nonlinear stress analysis (FORTRAN code for given *R/t*, *H*, *E*, σ_{ten} provided)
    - Adjust bending stress fraction for ρ
  - β_{ben} - from AFGROW

Options

- Table shows example of geometric nonlinearity parameter, $\rho = {}^{\sigma_{B_{NL}}}/_{\sigma_B}$ from nonlinear beam theory
- σ_T = 100MPa, $\sigma_B \neq 0$, E = 70 GPa

R/t	H=5	H=10	H=20	H=30	H=50	H=100	H=200
0.1	0.9995	0.9979	0.9916	0.9813	0.9494	0.8205	0.5074
0.2	0.9979	0.9916	0.9671	0.9285	0.8205	0.5074	0.1477
0.3	[illegible]	[illegible]	0.9285	0.8503	0.6604	0.2789	0.0405
0.4	0.9916	0.9671	0.8785	0.7576	0.5074	0.1477	0.0110
0.5	0.9869	0.9494	0.8205	0.6604	0.3791	0.0774	0.0030
0.6	0.9813	0.9285	[illegible]	[illegible]	0.2789	0.0405	0.0008
0.8	0.9671	0.8785	0.6284	0.4025	0.1477	0.0110	0.0001
1.0	0.9494	0.8205	0.5074	0.2789	0.0774	0.0030	0.0000
2.0	0.8205	0.5074	0.1477	0.0405	0.0030	0.0000	0.0000
3.0	0.6604	0.2789	0.0405	0.0058	[illegible]	[illegible]	0.0000
5.0	0.3791	0.0774	0.0030	0.0001	[illegible]	[illegible]	0.0000
10.0	0.0774	0.0030	0.0000	0.0000	0.0000	0.0000	0.0000

Option 2a

Option 2b

Option 1

Must calculate and assess ρ for each R/t, H, E, σ_{ten}

Conclusions

- Errors in 2003 *K* database are real
  - Numerical round-off error
  - Geometric nonlinearity
- Errors in very specific geometry not typically seen in aircraft structure
  - Affect seems minimal – no bug reports to date
- Using solutions as implemented is conservative
  - Over estimate *K* by 4 - 650% for examples shown
- Method provided for mitigating error and correcting for geometric nonlinearity

FORTRAN Code for ρ

```
 program RHOVALUES
c   use quadrupel precision for sinh cosh
      implicit real*16 (a-h,o-z)
      dimension RtValues(12),Harea(7),result(12,7)
      data RtValues/ 0.1, 0.2, 0.3, 0.4, 0.5,
     *              0.6, 0.8, 1.0, 2.0, 3.0,
     *              5.0,10.0/
      data Harea/ 5.0, 10.0, 20.0, 30.0, 50.0, 100.0,
     *200.0/
c      write(6,*) 'Give R/t and H'
c      read(5,*) Rt,H
c........calculate data in the table
      do 888 i=1,12
          Rt=Rtvalues(i)
      do 888 j=1,7
          H=Harea(j)
      thick=2.0D0/Rt
c      E=210000, steel
      E=71000.0
c    moment of inertia I,  (b=1)
      b=1.d0
      reall=b*thick**3/12.d0
c....determine the bending moment
        sigbend=30.D0
c       sigbend=M*(thick/2)/I
c       M=2*I*sigbend/thick
       realM=2.0*reall*sigbend/thick
c...........dragkraft
      sigtens=100.D0
      P=thick*sigtens
c........parameter n (KTH FS), gamma
      realn   =  sqrt(P/(E*reall))
      reall   =  2.0D0*H
      gamma   =  realn*reall
c    calculate displacement in the middle
      factor=((qcosh(gamma) - 1.d0)/qsinh(gamma))
      w=(realM/P)*(qcosh(realn*H)-factor*qsinh(realn*H) -1.0)
c......calculate w''
      wbiss=(-realM-P*w)/(E*reall)
      sigmacrack=-E*thick*wbiss/2.0
      rho=sigmacrack/sigbend
      write(6,120) Rt,H,w,sigmacrack,rho
 120    format('R/t=',f5.1,' H=',f5.1,' w(H)=',f12.6,'
Sigma_Crack',
 *f12.6,' Rho=',f15.6)
      result(i,j)=rho
 888    continue
          write(6,125) (Harea(j),j=1,7)
 125        format(4x,7f12.0)
      do 889 i=1,12
          write(6,130) Rtvalues(i),(result(i,j),j=1,7)
 130        format(f5.1,7f12.4)
 889    continue
      end
```